BAHAN AJAR

BAHASA INGGRIS MATEMATIKA

(ENGLISH FOR MATHEMATICS)

PALUPI SRI WIJAYANTI, M.PD.

NIS. 19890615 201508 2 010

PROGRAM SARJANA PENDIDIKAN MATEMATIKA

FAKULTAS KEGURUAN DAN ILMU PENDIDIKAN

UNIVERSITAS PGRI YOGYAKARTA

2020

FOREWORD

Praise our gratitude to the presence of Allah SWT for the outpouring of His compassion as well as for all the ease and fluency He provides, so that this teaching material can be completed properly.

This teaching material can be used as a student handbook in the English mathematics course. This teaching material was completed with the help of many parties, which is why we would like to express our appreciation and gratitude to:

1. Dear Chancellor and Vice Chancellor, Dean and Deputy Dean, head of the study program and all structural officials at the PGRI Yogyakarta University who have been resource persons and provided input regarding the material we need in the preparation of this teaching material.
2. Dear Educational Development Institute, PGRI Yogyakarta University who has provided opportunities and supported the writing of this teaching material.
3. Parents, siblings, relatives, and friends who have motivated us to always try our best in completing the writing of this teaching material.
4. Other parties who have helped the completion of this book that cannot be written down one by one.

To all the parties above, hopefully it will become amal Jariyah and get abundant rewards from Allah SWT, and may Allah SWT. always bestows protection, grace and guidance for all of us.

There is no ivory that is not cracked. We realize that this teaching material is far from perfect. Therefore we apologize for any shortcomings in this writing. We also hope that suggestions and criticism from readers can make this teaching material even better.

Thus the writing of this teaching material, hopefully it will be the first step to compile and develop teaching materials about English for teaching mathematics in the future. Hopefully this will be especially useful for the academics of the PGRI Yogyakarta University and all parties who have a concern for education in the State of Indonesia.

Yogyakarta, 24 Februari 2020

Penulis

TABLE OF CONTENT

BAB I.
SYMBOL AND MATHEMATICAL TERMS

A. INTRODUCTION

Teaching mathematics in English, it is necessary to know and understand mathematical terms and symbols. Symbols in mathematics sometimes have the same pronunciation in Indonesian, but sometimes there are differences. Mathematical symbols and terminology can be confusing and can be a barrier to learning and understanding basic numeracy.

B. MATERIAL

1. Symbols

a. Basic Math Symbols

Symbol	Symbol Name	Meaning / definition	Example
=	equals sign	used between two expressions to indicate that they take the same value.	$5 = 2+3$
≠	not equal sign	used between two expressions to indicate that they take the not same/different value	$5 \neq 4$
>	strict inequality	greater than	$5 > 4$
<	strict inequality	less than	$4 < 5$
≥	inequality	greater than or equal to	$5 \geq 4$
≤	inequality	less than or equal to	$4 \leq 5$
( )	parentheses	calculate expression inside first	$2 \times (3+5) = 16$
[ ]	Brackets	calculate expression inside first	$[(1+2)(1+5)] = 18$
+	plus sign	addition	$1 + 1 = 2$
−	minus sign	subtraction	$2 - 1 = 1$
±	plus - minus	both plus and minus operations	$3 \pm 5 = 8$ and -2
∓	minus - plus	both minus and plus operations	$3 \mp 5 = -2$ and 8
*	asterisk	multiplication	$2 * 3 = 6$
×	times sign	multiplication	$2 \times 3 = 6$
·	multiplication dot	multiplication	$2 \cdot 3 = 6$
÷	division sign / obelus	division	$6 \div 2 = 3$

/	division slash	division	$6 / 2 = 3$
–	horizontal line	division / fraction	$\frac{6}{2} = 3$
mod	modulo	remainder calculation	7 mod 2 = 1
.	period	decimal point, decimal separator	2.56 = 2+56/100
a^b	power	Exponent	$2^3 = 8$
a^b	caret	Exponent	2 ^ 3 = 8
$\sqrt{a}$	square root	$\sqrt{a} \cdot \sqrt{a} = a$	$\sqrt{9} = \pm 3$
$\sqrt[3]{a}$	cube root	$\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a$	$\sqrt[3]{8} = 2$
$\sqrt[4]{a}$	fourth root	$\sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} = a$	$\sqrt[4]{16} = \pm 2$
$\sqrt[n]{a}$	n-th root (radical)		for n=3, $\sqrt[n]{8} = 2$
%	percent	1% = 1/100	10% × 30 = 3
‰	per-mille	1‰ = 1/1000 = 0.1%	10‰ × 30 = 0.3
ppm	per-million	1ppm = 1/1000000	10ppm × 30 = 0.0003
ppb	per-billion	1ppb = 1/1000000000	10ppb × 30 = 3×10^{-7}
ppt	per-trillion	1ppt = 10^{-12}	10ppt × 30 = 3×10^{-10}

b. Geometry symbols

Symbol	Symbol Name	Meaning / definition	Example
∠	angle	formed by two rays	∠ABC = 30°
∡	measured angle		∡ABC = 30°
∢	spherical angle		∢AOB = 30°
∟	right angle	= 90°	α = 90°
°	degree	1 turn = 360°	α = 60°
′	arcminute	1° = 60′	α = 60°59′
″	arcsecond	1′ = 60″	α = 60°59′59″
$\overleftrightarrow{AB}$	line	infinite line	
AB	line segment	line from point A to point B	
$\overrightarrow{AB}$	ray	line that start from point A	
$\overset{\frown}{AB}$	arc	arc from point A to point B	$\overset{\frown}{AB}$ = 60°

∭	triple integral	integration of function of 3 variables	
∮	closed contour / line integral		
∯	closed surface integral		
∰	closed volume integral		
$[a,b]$	closed interval	$[a,b] = \{x \mid a \leq x \leq b\}$	
(a,b)	open interval	$(a,b) = \{x \mid a < x < b\}$	
i	imaginary unit	$i \equiv \sqrt{-1}$	$z = 3 + 2i$
z^*	complex conjugate	$z = a+bi \rightarrow z^*=a-bi$	$z^* = 3 + 2i$
z	complex conjugate	$z = a+bi \rightarrow z = a-bi$	$z = 3 + 2i$
∇	nabla / del	gradient / divergence operator	$\nabla f(x,y,z)$
$\bar{x}$	vector		
$\hat{x}$	unit vector		
$x * y$	convolution	$y(t) = x(t) * h(t)$	
$\mathcal{L}$	Laplace transform	$F(s) = \mathcal{L}\{f(t)\}$	
$\mathcal{F}$	Fourier transform	$X(\omega) = \mathcal{F}\{f(t)\}$	
δ	delta function		
∞	lemniscate	infinity symbol	

C. SUMMARY

The concept of mathematics is completely based on the relationship between numbers and symbols. These math symbols are used to perform different mathematical operations and represent various mathematical concepts.And although students sometimes find it challenging to use complex math symbols to solve math problems, these symbols are used to refer math quantities and help in easy denotation. There are a lot of math symbols applied in any given mathematical concept ranging from simple addition and subtraction to complicated operations like integration. This is the main reason why students find math symbols confusing to understand and difficult to remember. Since there are so many notations that are important for students to understand, we have put down a list of common math symbols, their concise definition and how to use them.

D. EXERCISE

Give examples using the symbols below:

Symbol	Symbol Name	Meaning / definition	Give an Example
=	equals sign	equality	
≠	not equal sign	inequality	
>	strict inequality	greater than	
<	strict inequality	less than	
≥	inequality	greater than or equal to	
≤	inequality	less than or equal to	
( )	parentheses	calculate expression inside first	
[ ]	brackets	calculate expression inside first	
+	plus sign	addition	
−	minus sign	subtraction	
±	plus - minus	both plus and minus operations	
∓	minus - plus	both minus and plus operations	
*	asterisk	multiplication	
×	times sign	multiplication	
·	multiplication dot	multiplication	
÷	division sign / obelus	division	
/	division slash	division	
–	horizontal line	division / fraction	
mod	modulo	remainder calculation	
.	period	decimal point, decimal separator	
a^b	power	exponent	
a^b	caret	exponent	
$\sqrt{a}$	square root	$\sqrt{a} \cdot \sqrt{a} = a$	
$\sqrt[3]{a}$	cube root	$\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a$	
$\sqrt[4]{a}$	fourth root	$\sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} = a$	

BAB II
SENTENCES THAT ARE MOST USED FOR TEACHING IN CLASSROOM

A. INTRODUCTION

The task of a teacher is to guide, teach, and educate their students at school. A teacher must also establish good communication with their students, so that teaching and learning activities run smoothly.

This time, we will discuss a collection of sentences or expressions that teachers commonly say in class, which are conveyed in English. From this material, we will find out how to say sentences that teachers usually communicate in English or understand what our teachers generally say every day.

B. MATERIAL

1. Greetings

a. Good morning everybody/class. - *Selamat pagi semuanya / kelas.*

b. Good afternoon everyone/class. - *Selamat siang semua orang / kelas.*

c. How are you? - *Apa kabar?*

d. What's up? - *Ada apa?*

e. Have a nice day! - *Semoga harimu menyenangkan!*

f. Let us wrap up for today! - *Mari kita akhiri hari ini!*

2. Magic words

a. Please. - *Tolong "memperhalus permintaan."*

b. Thank you. – *Terimakasih.*

c. I am sorry. - *Saya meminta maaf.*

d. Excuse me. - *Saya meminta maaf "lebih sopan."*

e. No, thank you. - *Tidak, terimakasih*

f. Pardon. - *Maaf "meminta pengulangan apa yang telah dikatakan."*

g. I appreciate it. - *Saya mengapresiasinya.*

h. You are welcome. - *Terimakasih kembali.*

i. My pleasure. - *Dengan senang hati.*

3. Giving Instructions in class

a. Listen carefully. - *Dengarkan baik-baik.*

b. Attention Please! - *Dimohon perhatiannya!*

c. Look at me! - *Lihat saya!*

d. Listen to what your classmates are saying. - *Dengarkan apa yang teman-teman Anda katakan.*

a. Let's recap before we go. - *Mari rekap sebelum kita pergi.*
b. Can you summarize what I said? - *Dapatkah anda meringkas apa yang saya katakan?*
c. OK class, we will stop here. - *Kelas OK , kami akan berhenti di sini.*
d. We will continue next week. - *Kita akan terus minggu depan.*
e. Let's wrap it for this week. - *Mari kita mengulangi materi minggu ini .*
f. Do you have any questions before we end this class for today? - *Apakah anda memiliki pertanyaan sebelum kami mengakhiri kelas ini untuk hari ini?*
g. Any questions before we stop? - *Ada pertanyaan sebelum kita berhenti?*
h. Let's stop here. - *Mari kita berhenti di sini.*
i. Before we stop, tell me what have you learnt today. - *Sebelum kita berhenti , katakan padaku apa yang harus Anda pelajari hari ini.*
j. Since we finished early, how about we play a game? - *Karena kita selesai awal, bagaimana kalau kita bermain game?*
k. There are few minutes left, you can check your work again. - *Ada beberapa menit tersisa, Anda dapat memeriksa pekerjaan Anda lagi.*
l. Saved by the bell, we will continue next week. – *Bel telah berbunyi, kita akan teruskan minggu depan.*
m. No time left, let's continue in next class. - *Tidak ada waktu yang tersisa, mari kita lanjutkan di kelas berikutnya .*
n. Goodbye everyone! - *Selamat tinggal/berpisah semuanya!*
o. Have a nice day! - *Semoga harimu indah!*

C. SUMMARY

A teacher must prepare all learning materials when going into class. Ensure high self-esteem and a cheerful face when meeting students for the first time. The first impression pays for everything. Therefore it would be much better if, from the first meeting, the teacher ensures that students feel comfortable.

D. EXERCISE

1. Write a greeting sentence and the conclusion of the lesson
2. What is your sentence to ask students to answer the teacher's question?
3. How is the teacher's sentence to correct student work and encourage it?

ANSWER

1. Good morning everybody/class, Have a good day!
2. Has everyone submitted their tests?
3. Good luck for your exams

CHAPTER III
MATHEMATICS IN PRIMARY SCHOOL

A. INTRODUCTION

Learning mathematics in elementary school is inseparable from two things: the essence of mathematics itself and the nature of elementary school students. The characteristics of elementary school mathematics learning include learning mathematics using the spiral method, gradually learning mathematics, learning mathematics using the inductive method, adhering to the truth of consistency, should be meaningful.

In learning the meaning of rules, arguments are not given in the finished form. Instead of rules, students use discussions through examples inductively in elementary school, then proven deductively at the next level.

Of course, teaching mathematics in elementary schools is not as easy as we imagine. Apart from students whose mindsets are still in the concrete operational phase, students' abilities are also very diverse.

B. MATERIAL

1. Introducing whole number arithmetic

The following key ideas must be emphasised:

place value
our number system has a base of ten
the role of zero as a placeholder
the additive property of place value (that 245 represents 200, 40 and 5 added)
the multiplicative property of place value (that the value of each successive column on the endless base ten chain is ten times the previous i.e. hundred = 10 x ten, thousand = 10 x hundred, ten thousand = 10 x thousand etc)

2. Place value

Although we have introduced place value using MAB, you would not do so with children. Use unstructured proportional materials first, e.g., bundling of icy pole sticks and straws before structured proportional materials such as MAB. Finally, use non-proportional models such as the abacus and counters.

Begin by making bundles of ten (icy pole sticks etc) to show the structure of numbers such as 23 from 2 bundles of ten and 3 more. In preparation for arithmetic algorithms, play lots of trading games. Make sure the activities include both the composition and decomposition of numbers, e.g., bundling (ones and tens) and unbundling (tens and ones). It is important to establish the notion that numbers can be represented in several ways, so that 245, for example, is 200 + 40 + 5 or 200 + 45 etc. Number expanders are excellent to demonstrate this and show possibilities.

- a million grains of rice (because 1 million is a big number and there are a lot of grains of rice in the bag - this naive strategy needs improvement)
- I think I can hold about a thousand grains of rice in my hand and there looks like about 10 handfuls in that bag, so there are are 10000 grains of rice in the bag - this strategy can be refined by counting the number of grains in a handful, or half a handful.
- I know that there are 1000 grams in a kilogram and I think each grain of rice weighs a hundredth of a gram so there must be 1000 times 100 grains of rice in the bag which equals 100000 grains of rice. This strategy can be refined by weighing a convenient number of grains (e.g. 50 or 100 grains) so the estimate is based on firmer information.

Estimation also depends on children having a good sense of the size of units of measurement and knowing other "benchmark" data (e.g. a big pace is about 1 metre, my handspan is about 20 cm, a litre of water weighs a kilogram).

C. SUMMARY

An integer is a number that consists of three types of numbers, namely negative numbers, zero and positive numbers. In arithmetic operations, there are several operations, starting from addition and subtraction. The next arithmetic operations are multiplication and division. The first characteristic of both count operations is that positive times positive, or negative times negative, the result is positive. A negative number divided by a negative and a negative number times a negative number also results in a positive number. The third property is positive numbers times negative equals negative, as well as division. The last property is a negative number times a positive number the result is negative, and a negative number divided by a positive number, the result is negative. Of the four properties, it is known that two numbers have the same sign, so the result is a positive number. Meanwhile, numbers with different signs will result in negative numbers. In this mix calculation operation, there is a class ladder, which consists of brackets at the top, a divide and times below it, and negative and positive signs at the bottom of the ladder. The sequence of symbols on this mixed count shows the way or series of operations, friends. That is, every single ladder, we work from the left. So if there are various signs in a problem, we must do it from the parentheses first, according to the order on the class ladder.

D. EXERCISE

1. Kim can walk 4 kilometers in one hour. How long does it take Kim to walk 18 kilometers?
2. A factory produced 2300 TV sets in its first year of production. 4500 sets were produced in its second year and 500 more sets were produced in its third year than in its second year. How many TV sets were produced in three years?
3. Linda bought 3 notebooks at $1.20 each; a box of pencils at $1.50 and a box of pens at $1.70. How much did Linda spend?
4. A large box contains 18 small boxes and each small box contains 25 chocolate bars. How many chocolate bars are in the large box?
5. It takes John 25 minutes to walk to the car park and 45 to drive to work. At what time should he get out of the house in order to get to work at 9:00 a.m.?
6. Tom and Bob have a total of 49 toys. If Bob has 5 more toys than Tom, how many toys does each one have?
7. John can eat a quarter of a pizza in one minute. How long does it take John to eat one pizza and a half?
8. John can eat a sixth of a pizza in two minutes. It takes 3 minutes for Billy to eat one quarter of the same pizza. If John and Billy start eating one pizza each, who will finish first?
9. Jim, Carla and Sammy are members of the same family. Carla is 5 years older than Jim. Sammy is 6 years older than Carla. The sum of their three ages is 31 years. How old is each one them?
10. John read the quarter of the time that Tom read. Tom read only two-fifth of the time that Sasha read. Sasha read twice as long as Mike. If Mike read 5 hours, how long did John read?

Answer (Jawaban)

1. 4 hours and 30 minutes
2. 11,800 TV sets
3. $6.80
4. 450 chocolate bars
5. 7:50 a.m.
6. Tom: 22 , Bob: 27
7. 6 minutes
8. same time , 12 minutes

9. Jim: 5 years , Carla: 10 years , Sammy: 16 years

10. 1 hour

CHAPTER IV
ALGEBRA IN YUNIOR HIGH SCHOOL

A. INTRODUCTION

Algebraic material learning objectives based on the 2013 curriculum

Mathematics for the VIII grade of SMP / MTs, including: (1) aspects of attitude; through observation, question and answer, group discussion, students can show a sense of desire know, have confidence, and have an interest in understanding algebraic material; (2) aspects knowledge; through oral and written tests students can complete brief descriptions algebraic material; (3) skills aspects; through independent and group assignments, students can solve problems related to algebra material.

As for expected student learning experiences after learning algebra (Teacher's Book Mathematics VIII, 2014: 29): (1) Students can apply algebraic operations involves rational numbers in symbolic problems; (2) Students able to use algebraic operations involving rational numbers to problems verbal. While the coverage of algebra material (Mathematics Teacher Book VIII, 2014: 40) namely: (1) Forms and Elements of Algebra, including forms and definitions of algebraic terms, algebraic elements (variables, coefficients, constants, powers) and like terms; (2) Algebraic operations, including addition, subtraction, multiplication, division and rank; (3) simplification of algebraic forms, and (4) problem solving.

B. MATERIAL

Solving Equations

After explaining the basic concepts in solving equations, I show students a simple equation to solve such as $2x + 4 = 8$. I tell students that the goal is to get x by itself on one side of the equation. So the first step is to subtract 4 from both sides, and we are left with $2x = 4$. Most are able to follow this procedure. Then when I ask them what should be the next step, some students want to subtract the 2 from the $2x$. So I explain that $2x$ means 2 multiplied by x, so to get x by itself we need to divide by 2. This satisfies some students but others remain confused.

I have achieved good results by turning this equation into a word problem: Two cantaloupes = \$4. I ask "How much is one cantaloupe?" and they all answer \$2. Then I ask "How did you figure this out?" This helps them realize that they had to divide both sides by 2 in order to get the answer.

In order to find the LCM of 12, 15 and 18, break each number down into its prime factors:

12: 3,2,2
15: 3,5
18: 3,3,2
So the LCM is 3 x 3 x 2 x 2 x 5 = 180

To find the LCM of $x^2 - 4$, $x^2 + 4x + 4$, $2x - 4$, break each expression into its prime factors (after defining a prime factor for an algebraic expression):

$x^2 - 4$: $(x + 2)(x - 2)$
$x^2 + 4x + 4$: $(x + 2)(x + 2)$
$2x - 4$: $2(x - 2)$

Using the same procedure we used for numbers, we pick each prime factor to the maximum number of times it appears in any of the three expressions:

So the LCM is $2(x + 2)(x + 2)(x - 2)$

As opposed to with numbers, we don't usually multiply out all the terms together.

What is the Meaning of "=" (Equals)?

In one lecture on the distributive property I wrote the following equation on the board: $2(x + 3) = 2x + 6$

One student asked "But what is the answer?" I did not at first understand what the student was getting at, and I asked her to explain. She said she was told in her previous math classes in high school that equations always have an answer, so what is the answer here? Then I realized that "equals" has actually two meanings. In one case if I write $x + 2 = 6$ then the answer is $x = 4$. But in the equation above, the right side is a transformation of the left side—a transformation into an equivalent but differently formatted expression. Since then, I always make my students aware of the different uses of the term "equals."

C. SUMMARY

Algebra is an essential branch of mathematics that is often considered difficult and abstract (Laila Hayati, 2013: 398). Thinking algebra, a student must be able to understand patterns, relationships and functions, represent and analyze Mathematical situations and structures using algebraic symbols, using mathematical models for meaning and understanding quantitative relationships, and exploring change in multiple contexts. One of the obstacles in algebra is to declare an expression using symbols. Standard algebra emphasizes the relationship between quantities, including functions, representing mathematical relationships and change analysis. Functional relationships can be expressed using symbolic notation. Algebraic thinking is an essential and fundamental element of mathematical thinking and reasoning skills. One way to develop students' thinking skills is to develop students' algebraic thinking skills by familiarizing them with solving problems. An essential aspect of algebraic thinking is the ability to consider the relatedness and generalizations of situations where if it can understand conceptions, students' abilities can develop. Algebraic thinking is based on fundamental mathematical ideas and concepts, and in turn, these ideas are used to solve one's problem increasingly sophisticated. The elements in algebraic form are terms. The term can be a constant, a variable or product/power, drawing the continuous root of variables, but not their additions. So, each term is a form of algebra that is simpler than the more complex algebra forms.

D. EXERCISE

Solve each inequality, graph each solution, and give interval notation.

1. $-47>8-5x$
2. $-2(3+k)<-44$
3. $18<-2(-8+p)$
4. $24>-6(m-6)$
5. $-r-5(r-6)<-18$

ANSWER

1. $x>11$: $[11,\infty)$
2. $k>19$: $(19,\infty)$
3. $p<-1$: $(-\infty,-1)$
4. $m>2$: $[2,\infty)$
5. $r>8$: $(8,\infty)$

CHAPTER V
TEACHING AND LEARNING TRYGONOMETRY IN SENIOR HIGH SCHOOL

A. INTRODUCTION

Consider a situation in which you are building a ramp so that people in wheelchairs can access a building. If the ramp must have a height of 8 feet, and the angle of the ramp must be about 5°, how long must the ramp be? Solving this kind of problem requires trigonometry. Recall that in the first lesson, you learned that the word trigonometry comes from two words meaning *triangle* and *measure*. In this lesson we will define six trigonometric functions. For each of these functions, the elements of the domain are angles. We will define these functions in two ways: first, using right triangles, and second, using angles of rotation. Once we have defined these functions, we will be able to solve problems like the one above.

B. MATERIAL

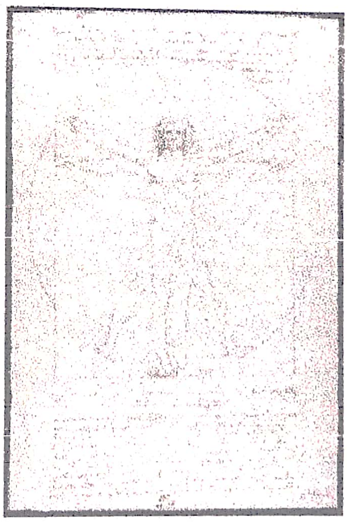
Motivation: Trig Is Anatomy

Imagine Bob The Alien visits Earth to study our species.

Without new words, humans are hard to describe: "There's a sphere at the top, which gets scratched occasionally" or "Two elongated cylinders appear to provide locomotion".

After creating specific terms for anatomy, Bob might jot down typical body proportions:

- The armspan (fingertip to fingertip) is approximately the height
- A head is 5 eye-widths wide
- Adults are 8 head-heights tall



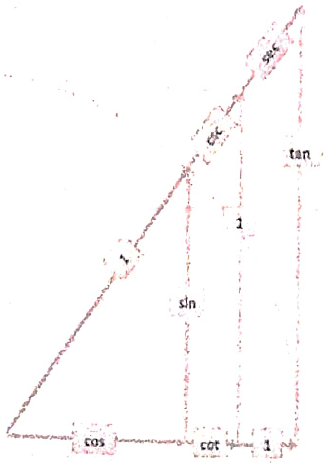
How is this helpful?

This is how we find out "sine/cosine = tangent/1".

I'd always tried to memorize these facts, when they just jump out at us when visualized. SOH-CAH-TOA is a nice shortcut, but get a real understanding first!

Gotcha: Remember Other Angles

Psst... don't over-focus on a single diagram, thinking tangent is always smaller than 1. If we increase the angle, we reach the ceiling before the wall:



The Pythagorean/similarity connections are always true, but the relative sizes can vary. (But, you might notice that sine and cosine are always smallest, or tied, since they're trapped inside the dome. Nice!)

Summary: What Should We Remember?

For most of us, I'd say this is enough:

- Trig explains the anatomy of "math-made" objects, such as circles and repeating cycles
- The dome/wall/ceiling analogy shows the connections between the trig functions
- Trig functions return percentages, that we apply to our specific scenario

You don't need to memorize 12+cot2=csc2, except for silly tests that mistake trivia for understanding. In that case, take a minute to draw the dome/wall/ceiling diagram, fill in the labels (a tan gentleman you can see, can't you?), and create a cheatsheet for yourself. In a follow-up, we'll learn about graphing, complements, and using Euler's Formula to find even more connections.

Not too bad, right? Before the dome/wall/ceiling analogy, I'd be drowning in a mess of computations. Visualizing the scenario makes it simple, even fun, to see which trig buddy can help us out.

In your problem, think: am I interested in the dome (sin/cos), the wall (tan/sec), or the ceiling (cot/csc)?

C. SUMMARY

The right-angled triangle is the star of trigonometry because it is mostly the triangle of interest. So, why study trig? Honestly, this aspect of mathematics can be difficult and confusing at first but you will appreciate the versatility when you grasp it. Moreover, the study helps us to find angles & distances very useful in engineering, animations, computer games and so on.

When you hear the mnemonics SOH-CAH-TOA , what comes to your mind? Firstly, trigonometric ratios: Sine, Cosine & Tangent directly represent the relationship between two sides of a right-angled triangle. In this manner, each ratio represents one side divided by the other. So when next you are asked, trigonometric ratios are simply one side of a right-angled triangle divided by another is not out of place. In fact, you are 100% right! For instance, Sine represents the ratio of the side called opposite to the hypotenuse (longest side). Secondly, inverse trigonometric ratios: Co-secant, Secant and Cotangent are reciprocals of the three basic trigonometric ratios Sine, cosine and tangent respectively. Along this line, each inverse is one (1) divided by a trigonometric ratio.

D. EXERCISE

1. Prove the identity: $\tan^2(x) - \sin^2(x) = \tan^2(x)\sin^2(x)$
2. Prove the identity: $4\sin(x)\cos(x) - \sin(4x) / \cos(2x)$
3. Solve the trigonometric equation given by: $\sin(x) + \sin(x/2) = 0$ for $0 \leq x \leq 2$ pi
4. Solve the trigonometric equation given by: $(2\sin(x) - 1)(\tan(x) - 1) = 0$ for $0 \leq x \leq 2$ pi
5. Solve the trigonometric equation given by: $\cos(2x)\cos(x) - \sin(2x)\sin(x) = 0$ for $0 \leq x \leq 2$ pi.

Answer

1. Use the identity $\tan(x) = \sin(x) / \cos(x)$ in the left hand side of the given identity.

$\tan^2(x) - \sin^2(x) = \sin^2(x) / \cos^2(x) - \sin^2(x)$

$= [\sin^2(x) - \cos^2(x)\sin^2(x)] / \cos^2(x)$

$= \sin^2(x)[1 - \cos^2(x)] / \cos^2(x)$

$= \sin^2(x)\sin^2(x) / \cos^2(x)$

$= \sin^2(x)\tan^2(x)$ which is equal to the right hand side of the given identity.

2. Use the identity sin(2x) = 2 sin(x) cos(x) to write sin(4x) = 2 sin(2x) cos(2x) in the right hand side of the given identity.

sin(4x) / cos(2x)

= 2 sin(2x) cos(2x) / cos(2x)

= 2 sin(2x)

= 2 [2 sin(x) cos(x)]

= 4 sin(x) cos(x) which is equal to the right hand side of the given identity.

3. Use the identity sin(2x) = 2 sin(x) cos(x) to write sin(x) as sin(2 * x/2) = 2 sin(x / 2) cos(x / 2) in the right hand side of the given equation.

2 sin(x / 2) cos(x / 2) + sin(x / 2) = 0

sin(x/2) [2 cos(x/2) + 1] = 0 factor

which gives

sin(x/2) = 0 or 2 cos(x/2) + 1 = 0

sin(x / 2) = 0 leads to x / 2 = 0 or x / 2 = Pi which leads to x = 0 or x = 2pi

2 cos(x/2) + 1 = 0 leads to cos(x/2) = -1/2 which leads to x/2 = 2pi/3 and x/2 = 4pi/3

(the second solution leads to x greater than 2pi)

solutions: x = 0, x = 4pi/3 and x = 2pi

4. The given equation is already factored

(2sin(x) - 1)(tan(x) - 1) = 0

which means

2sin(x) - 1 = 0 or tan(x) - 1 = 0

sin(x) = 1/2 or tan(x) = 1 equivalent equations to the above

solutions: x = pi/6, 5pi/6, x = pi/4 and x = 5pi/4

5. Note that cos(2x + x) = cos(2x) cos(x) - sin(2x) sin(x) using the formula for cos(A + B). Hence cos(2x) cos(x) - sin(2x) sin(x) = 0 is equivalent to cos(3x) = 0

Solve for 3x to obtain: 3x = pi/2, 3x = 3Pi/2, 3x = 5pi/2, 3x = 7pi/2, 3x = 9pi/2 and 11pi/2

solutions: x = pi/6, pi/2, 5pi/6, 7pi/6, 3pi/2 and 11pi/6

CHAPTER VI.
LEARN CALCULUS IN HIGHER EDUCATION

A. PENDAHULUAN

Calculus is a branch of mathematics containing limits, derivatives, integrals and functions. Calculus is a major part of mathematics. Calculus is used in mechanical, physics etc. For most of the students calculus is hardest part of mathematics. Are you struggling to understand the calculus? Calculus can can be easy if approached in a proper way. Follow the article to learn calculus in the right manner.

B. PENYAJIAN

In the previous **section** we looked at a couple of problems and in both problems we had a function (slope in the tangent problem case and average rate of change in the rate of change problem) and we wanted to know how that function was behaving at some point x=ax=a. At this stage of the game we no longer care where the functions came from and we no longer care if we're going to see them down the road again or not. All that we need to know or worry about is that we've got these functions and we want to know something about them.

To answer the questions in the last section we choose values of xx that got closer and closer to x−ax−a and we plugged these into the function. We also made sure that we looked at values of xx that were on both the left and the right of x=ax=a. Once we did this we looked at our table of function values and saw what the function values were approaching as xx got closer and closer to x=ax=a and used this to guess the value that we were after.

This process is called **taking a limit** and we have some notation for this. The limit notation for the two problems from the last section is,

limx→12−2x2x−1=−4limt→5t3−6t2+25t−5=15limx→1 2−2x2x−1=−4limt→5 t3−6t 2+25t−5=15

In this notation we will note that we always give the function that we're working with and we also give the value of xx (or tt) that we are moving in towards. In this section we are going to take an intuitive approach to limits and try to get a feel for what they are and what they can tell us about a function. With that goal in mind we are not going to get into how we actually compute limits yet. We will instead rely on what we did in the previous section as well as another approach to guess the value of the limits.

This function clearly does not settle in towards a single number and so this limit **does not exist!**

This last example points out the drawback of just picking values of the variable and using a table of function values to estimate the value of a limit. The values of the variable that we chose in the previous example were valid and in fact were probably values that many would have picked. In fact, they were exactly the same values we used in the problem before this one and they worked in that problem!

When using a table of values there will always be the possibility that we aren't choosing the correct values and that we will guess incorrectly for our limit. This is something that we should always keep in mind when doing this to guess the value of limits. In fact, this is such a problem that after this section we will never use a table of values to guess the value of a limit again.

This last example also has shown us that limits do not have to exist. To that point we've only seen limits that existed, but that just doesn't always have to be the case.

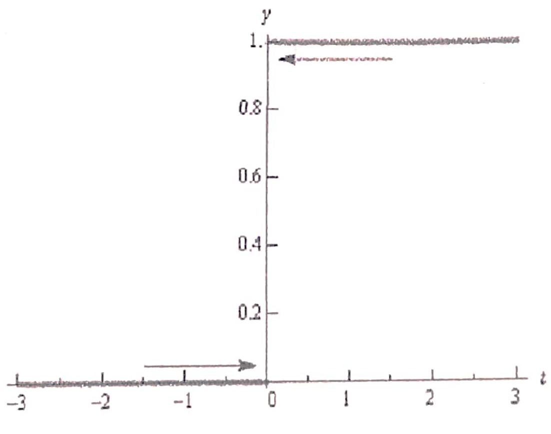
Let's take a look at one more example in this section.

Example 5 Estimate the value of the following

limit.limt→0H(t)where,H(t)={0if t<01if t≥0limt→0 H(t)where,H(t)={0if t<01if t≥0

Hide Solution

This function is often called either the **Heaviside** or **step** function. We could use a table of values to estimate the limit, but it's probably just as quick in this case to use the graph so let's do that. Below is the graph of this function.



We can see from the graph that if we approach t=0t=0 from the right side the function is moving in towards a yy value of 1. Well actually it's just staying at 1, but in the terminology that we've been using in this section it's moving in towards 1...

limit you need to be able to actually sketch the graph. For many functions this is not that easy to do.

There is another drawback in using graphs. Even if you have the graph it's only going to be useful if the yy value is approaching an integer. If the yy value is approaching say −15123−15123 there is no way that you're going to be able to guess that value from the graph and we are usually going to want exact values for our limits.

So, while graphs of functions can, on occasion, make your life easier in guessing values of limits they are again probably not the best way to get values of limits. They are only going to be useful if you can get your hands on it and the value of the limit is a "nice" number.

The natural question then is why did we even talk about using tables and/or graphs to estimate limits if they aren't the best way. There were a couple of reasons.

First, they can help us get a better understanding of what limits are and what they can tell us. If we don't do at least a couple of limits in this way we might not get all that good of an idea on just what limits are.

The second reason for doing limits in this way is to point out their drawback so that we aren't tempted to use them all the time!

We will eventually talk about how we really do limits. However, there is one more topic that we need to discuss before doing that. Since this section has already gone on for a while we will talk about this in the next section.

C. SUMMARY

Calculus, originally called **infinitesimal calculus** or "the calculus of infinitesimals", is the mathematical study of continuous change, in the same way that geometry is the study of shape and algebra is the study of generalizations of arithmetic operations.

It has two major branches, differential calculus and integral calculus; the former concerns instantaneous rates of change, and the slopes of curves, while integral calculus concerns accumulation of quantities, and areas under or between curves. These two branches are related to each other by the fundamental theorem of calculus, and they make use of the fundamental notions of convergence of infinite sequences and infinite series to a well-defined limit.[1]

Infinitesimal calculus was developed independently in the late 17th century by Isaac Newton and Gottfried Wilhelm Leibniz.[2][3] Today, calculus has widespread uses in science, engineering, and economics.[4]

In mathematics education, *calculus* denotes courses of elementary mathematical analysis, which are mainly devoted to the study of functions and limits. The word *calculus* (plural *calculi*) is a Latin word, meaning originally "small pebble" (this meaning is kept in medicine – see Calculus (medicine)). Because such pebbles were used for calculation, the meaning of the word has evolved and today usually means a method of computation. It is therefore used for naming specific methods of calculation and related theories, such as propositional calculus, Ricci calculus, calculus of variations, lambda calculus, and process calculus.

D. EXERCISE

2. Let m be the slope of the tangent line to the graph of $y = \dfrac{x^2}{x+2}$ at the point $(-3, -9)$. Express m as a limit. *(Do not compute m.)*

Solution: The slope m of the tangent line to the graph of $y = f(x)$ at the point $(x_0, f(x_0))$ is given by the limit

$$m = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

or equivalently by the limit

$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

Therefore two possible answers are

$$m = \lim_{x \to -3} \frac{\frac{x^2}{x+2} - (-9)}{x - (-3)} = \lim_{h \to 0} \frac{\frac{(-3+h)^2}{-3+h+2} - (-9)}{h}$$

□

4. Suppose that $\lim_{x \to c} f(x) = 0$ and there exists a constant K such that $|g(x)| \le K$ for all $x \ne c$ in some open interval containing c. Show that $\lim_{x \to c} (f(x)g(x)) = 0$.

Solution: We have $|f(x)g(x)| = |f(x)| \cdot |g(x)| \le |f(x)| \cdot K$ in some open interval around c. Therefore

$$-K|f(x)| \le f(x)g(x) \le K|f(x)|$$

Now applying the Sandwich Theorem and using the fact that $\lim_{x \to c} |f(x)| = 0$ we obtain the result. □

Suppose that $\lim_{x \to a} f(x) \neq 0$ and $\lim_{x \to a} g(x) = 0$. Show that $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist.

Solution: Assume that $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists, and let $L = \lim_{x \to a} \frac{f(x)}{g(x)}$. Then by the product rule for limits we obtain

$$\lim_{x \to a} f(x) = \lim_{x \to a} \left(\frac{f(x)}{g(x)} \cdot g(x) \right) = \lim_{x \to a} \frac{f(x)}{g(x)} \cdot \lim_{x \to a} g(x) = L \cdot 0 = 0 .$$

This contradicts the fact that $\lim_{x \to a} f(x) \neq 0$. Therefore our assumption cannot be true: $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist. □

DAFTAR PUSTAKA

Borg & Gall. (1979). *Educational Research: an introduction*. New York: Longman, Inc.

Clark, John & Holton, Derek Allan. (1991). *A First Look At Graph Theory*. Singapore: World Scientific Publishing Co. Pte. Ltd, 198.

Cropley, A.J. (2001). Creativity & cognition: producing effective novelty. *Roeper Review, 21*, 253-263.

George Booker dan Denise Bond. 2004. *Teaching Mathematics*. Melbourne: Pearson Prentice Hill.

Hamzah Upu. 2009. *English for Mathematics Instruction*. Makasar: Penerbit Refleksi.

Lie, S. (2006). *Mathematics education in diferent cultural traditions: A comparative study of East Asia and the West*. New York: Springer.

Marton, F., & Saljo, R. (1976). *The experiences of learning*. Edinburg, UK: Scottish Academy Press.

Rudy Hariyoo. 2009. *Complete English Grammar*. Gita Media: Jakarta.

Tamzen Armer. 2011. *Cambridge English for Scientists*. Cambridge University Press: New York.