CONSTRUCTION OF LEARNING TRAJECTORIES FOR ELEMENTARY SCHOOL STUDENTS IN THE CONTEXT OF PRAMBANAN TEMPLE

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Abstract

This research aims to describe the Learning Trajectory of elementary school students in multiplication and division activities in the context of Prambanan Temple. The research method used is qualitative with didactical design research approach, namely, (1) Prospective Analysis (determining learning obstacles); (2) Preparation of Hypothetical Learning Trajectory (HLT); (3) Pilot Experiment; (4) Teaching Experiment; and (5) Retrospective Analysis. The research was conducted on third grade students in 2 elementary schools in Yogyakarta. The number of participant groups was 14 students and 17 students, respectively. The results of the teaching experiment show that the LEGO building activity can increase students' understanding of the concept of multiplication. Meanwhile, the activity of sticking LEGO stickers using a fair distribution process can foster students' number sense. The resulting learning trajectory, according to the results of the retrospective analysis, shows several stages of learning, namely, (1) Situational, (2) Referential, (3) Practical, (4) Mathematical, and (5) Formal.

Keyword: Didactic; LEGO; Learning Trajectory; Mathematical Tasks; Prambanan temple

A. INTRODUCTION

Numeracy is a person's ability to use mathematical content to solve problems in life contexts (Perso 2006; Tout 2020). According to Lusardi (2012); & Purpura et al. (2011), numeracy ability is a crucial ability to become essential human capital for facing life in the future. Furthermore, according to Tariq (2014), numeracy skills can also be a steppingstone for someone to open opportunities to gain more complex scientific insights in the future. Seeing the importance of numeracy skills, it is necessary to get used to providing context in learning mathematics in elementary schools as the earliest educational institution for a person.

Even though awareness that it is crucial to train numeracy skills in elementary schools has emerged teachers still have difficulty using context in teaching mathematics. The research results of Askew et al. (1997); & Meeks, Kemp, and Stephenson (2014) show that teachers often need to use precise contexts in instilling mathematical concepts in elementary schools. Teachers often immediately provide mathematical formulas. This has an impact on students' numeracy skills, which need to be revised. This is supported by the research results of Catalano (2014); Segers, Kleemans, and Verhoeven (2015); & Wright (2013), that students' numeracy abilities are still meager. Students find it challenging to understand problems that have context because they are used to being given mathematical formulas directly by the teacher.

Previous research related to numeracy focused on the level of ability of teachers and students to solve problems according to context. Apart from that, there are also research results developed by Gittens (2015), where the use of mathematical tasks in the context of everyday life can improve students' numeracy skills. Furthermore, there is also the development of level-based worksheet-based learning media by Lakhsman (2019), which can also improve the numeracy skills of students in elementary schools. Different from the previous one, this research will use a mathematical task tool that has a cultural context to improve the numeracy skills of elementary school students. This research also uses a Didactical Design Research (DDR) approach, where researchers design mathematical tasks according to learning obstacle findings. This will make this research more focused on solving the problems of teachers' and students' difficulties in dealing with mathematical concepts. The learning obstacle findings show that the main problem faced by teachers and students is the concept of multiplication.

The use of cultural context has also never been done in previous studies. The cultural context was chosen because it is more exciting and valuable for students (Prahmana 2021). Cultural context can be of particular interest to students, and students become more curious when cultural context is used in mathematics learning (Susiana, Caswita, and Noer 2020). Apart from that, the cultural context can also provide benefits to students, where cultural values can provide additional insight that is useful for teaching positive character values to students (Cimen 2014). The cultural context used in this research is Prambanan Temple. The choice of Prambanan Temple considered the interlocking system used by researchers to teach the concept of multiplication in numeracy activities.

Based on the explanation above, the main objective of this research is to describe the learning trajectories of elementary school students in numeracy activities in the context of Prambanan Temple on multiplication material. The research questions in this study are:

1. What is the learning process using mathematical tasks in the context of Prambanan Temple?
2. What are the results of the description of the Learning Trajectory for elementary school students in numeracy activities in the context of Prambanan Temple?

B. LITERATURE REVIEW

1. Didactics

Historically, teaching theory can be reviewed back to the ancient Greek era, when Socrates gave an example of how Socrates taught. With indirect questions and directions, Socrates taught someone to find an understanding in their way (Hudson and Schneuwly 2007). Such a process is referred to in Greek as 'didaskein,' which means the formation of knowledge (knowledge formation) (Hopmann 2007). Based on experts' explanations regarding the definition of didactics, it can be underlined that didactics are all efforts that shape students' knowledge of a subject matter to make it easy for students to interact with the subject matter and understand the concepts of the subject matter well.

Didactics was developed in several theories; one of the well-known didactic theories is the Theory of Didactical Situation (TDS), which was discovered by Brousseau in the 1970s (Brousseau 2002). Brosseau developed the Theory of Didactical Situation (TDS) and examines the approach used to connect knowledge, in this case, school mathematics material, with a teaching method called the Theory of Didactical Situation. Brousseau explained that, in general, a teaching situation can be described as an interaction between students, teachers, and the environment (milieu). The discussion about TDS includes didactic situations, learning obstacles, learning trajectories, and didactic contracts.

2. Learning Obstacle

Carvalho et al (2004) said that the obstacles experienced by students through the learning process caused students to get less than optimal results. Then Vásquez (2015) said that obstacles are knowledge that can be used in solving various types of problems. However, if applied in the context of a new problem, this knowledge is insufficient or contradictory. There are three types of barriers to learning, according to Brousseau (2002), namely ontogenic, didactical, and epistemological:

1) Ontogenic Obstacle

Ontogenic barriers are obstacles that arise due to student limitations (from a neurophysiological perspective) during their development. Students develop knowledge that is appropriate to their abilities and goals at a certain age (Brousseau, 2002). Suryadi (2019) explains that ontogenic obstacles are a type of difficulty related to a child's readiness to learn. There are three types of learning difficulties in ontogenic obstacles, namely psychological, instrumental, and conceptual. The ontogenic psychological obstacle is a student's unpreparedness for learning, which is caused by psychological aspects, for example, motivation and interest or low interest in the material being studied. Ontogenic instrumental obstacles are children's technical difficulties, so that children cannot fully participate in learning because they do not understand the technical matters. Ontogenic conceptual obstacles are student difficulties related to the conceptual level contained in the design, which is less appropriate to the child's condition, as seen from previous learning experiences.

2) Didactical Obstacle

Didactic barriers are obstacles that originate from the education system or the presentation of a concept in the curriculum used (Brousseau, 2002). Didactical obstacles relate to the sequence or stages of presenting curriculum material (lack of detail or details), which have a significant impact on the learning process (Suryadi, 2019).

3) Epistemological Obstacle

Epistemological barriers are obstacles experienced by students in the process of acquiring knowledge. Epistemological barriers are caused by the limited context used when a concept is first studied (Brousseau, 2002). In other words, this obstacle occurs because of students' limited understanding of something related to a particular context according to their learning experience.

3. Learning Trajectory

Carvalho et al. (2004) define a learning trajectory as a description of children's thinking and learning in a particular domain and a particular flow of assumptions through a series of instructional tasks designed to foster mental processes or actions that are appropriate to the level of thinking and support goals in mathematics learning. Prahmana et al (2023) say that learning trajectories are the stages that students go through during the learning process to master the planned learning objectives. Learning Trajectory includes levels of understanding that are arranged to develop students' understanding from their existing level to a higher level. In addition, Sarama and Clements (2009) mention that learning trajectories are not a list of everything children need to learn; most importantly, they describe children's level of thinking, not just students' ability to answer math questions correctly.

According to Suryadi (2019), HLT is part of the mathematics teaching cycle, which connects the assessment of student knowledge, teacher knowledge, and hypothetical learning trajectories. HLT is a way to explain important aspects of the pedagogical thinking involved in teaching mathematics to understand how a mathematics educator (teacher, researcher, and curriculum developer), oriented by a constructivist perspective and the goals of mathematics learning for students, can think about the design and use of mathematics tasks to promote conceptual learning of mathematics.

Simon (2020) said that teachers provide direction for other components; the selection of learning tasks and hypotheses about students' learning processes are interrelated; tasks are selected based on hypotheses about the learning process; Learning process hypotheses are based on the tasks involved. In the end, Prahmana et al. (2023) summarize that HLT consists of three components, including learning objectives, a series of learning tasks, and a learning process that can be hypothesized, namely regarding predictions about how students' thinking and understanding will develop in a learning activity. This is in line with Oonk, Verloop, and Gravemeijer (2019), which states that HLT consists of three main components, including 1) Learning objectives for students, 2) Learning activities, and 3) Learning process conjecture about how to find out students' understanding and strategies that emerge when learning activities are carried out.

C. METHOD

1. Research Design

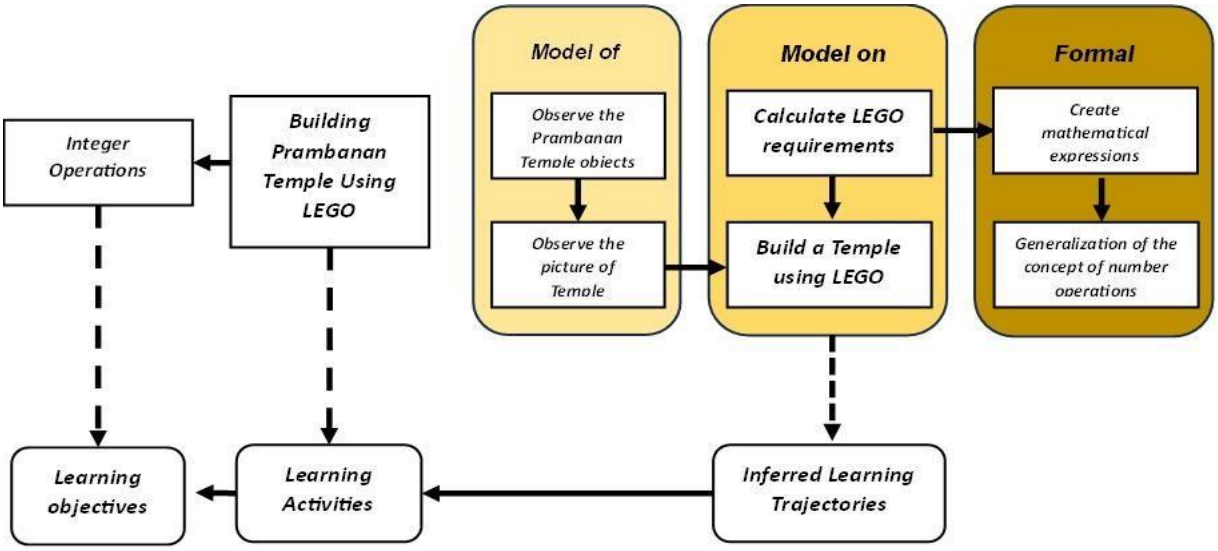
This research is qualitative, using a Didactical Design Research (DDR) research design. Didactical Design Research has a basis in two paradigms, namely the interpretive paradigm and the critical paradigm (Suryadi, 2019). The interpretive paradigm includes the study of reality phenomena related to the impact of didactic design on a person's way of thinking. The critical paradigm has the primary goal of implementing changes to existing didactic designs. The new didactic design will be developed as a step in improving learning stages to minimize the learning obstacles faced by students, especially regarding student numeracy.

2. Research Procedure

The research procedure begins with the process of exploring information on learning obstacles (prospective analysis). This data mining was carried out using a mathematical content perspective from the student and teacher perspective. After that, the researcher prepared a learning tool in the form of a mathematical assignment that contained numeracy activities with a cultural context. The Mathematical Task used in this research uses the Prambanan Temple context, which is assisted with LEGO as a learning aid. This Mathematical Task is also accompanied by a Hypothetical Learning Trajectory (HLT), which occurs to students while numeracy activities are in progress. The HLT used by researchers is as follows:

Figure 1.

HLT Concept of Multiplication and Division Number Operations for Students



After the HLT has been prepared, the researcher carries out the process of implementing the learning tools that have been prepared. Implementation was carried out in 2 meetings, with the researcher himself as the teacher in the class. After the learning process was complete, the researcher used retrospective analysis with a phenomenological approach to describe the learning trajectory that occurred during the implementation process.

3. Participant

The research subjects in this study were 14 third-grade students from X Elementary School and 18 third-grade students from Y Elementary School, where both schools are in Yogyakarta, Indonesia. All students will be subjects in the research, so there is no subject selection procedure. All parents of research subjects have signed a ***Letter of Consent*** indicating their willingness to participate in this research voluntarily.

4. Instruments and Data Collection

a. Learning Obstacle

To explore data related to learning obstacles, researchers used three instruments, namely, (1) in-depth interview guidelines for teachers and students, (2) learning observation sheets, and (3) numeracy test questions. The data collection process was carried out sequentially. First, the researcher conducted interviews with teachers and students separately. The interview process was carried out face-to-face to obtain more accurate information from the research subjects. The focus of this interview is information on what learning barriers students have and what teaching barriers experienced by teachers, especially related to mathematics content. After the interview process was complete, the researcher carried out observations in the classroom to confirm the results of the interviews that had been conducted previously. The researcher used two observers in the class, each of whom focused on teacher and student activities during learning. Finally, researchers gave numeracy test questions to students to see which numeracy content was the most difficult for students in class.

b. Mathematical Task

After the researcher obtained information related to the obstacles faced by students and teachers, the researcher prepared mathematics activities in the form of mathematics assignments. The results of prospective analysis show that the most difficult mathematics content for students and teachers is multiplication and division. Because of this, the researcher prepared mathematical activities related to the concept of number operations which can be seen in table 1 below:

Table 1.

Mathematical Task Number Operations

Real Situation	***Number of LEGOs given***
	Purple LEGO ➔ Thousand
	Blue LEGO ➔ Hundred
	Pink LEGO ➔ Dozens
	Green LEGO ➔ Unit

The Mathematical Task given to students consists of 3 activities, namely, (1) Arranging LEGO according to the Place Value concept; (2) Arranging LEGOs and counting the number of LEGOs using the concept of Multiplication; and (3) Disassembling LEGOs to divide the number of LEGOs equally using the division concept.

c. ***Retrospective Analysis***

For retrospective analysis, researchers used video recording instruments and observation sheets to record the learning process that occurred in the classroom. The observation process is carried out during learning with mathematical tasks. This analysis is carried out to compile a Learning Trajectory that occurs during the learning process. Apart from that, the results of this Learning Trajectory will be compared with the HLT that was previously prepared by the researcher.

5. Data

To maintain the validity of the data, researchers used several methods, including, (1) data triangulation; (2) Focus Group Discussion; (3) Cross-section data. Data triangulation was obtained from using 3 different instruments. To strengthen it, researchers also conducted FGDs with teachers and experts regarding research instruments, learning tools (mathematical tasks), and the Hypothetical Learning Trajectory prepared by researchers. Finally, the researcher also carried out cross-section data with school principals and teachers to ensure that the data obtained was in accordance with the teaching experience of principals and teachers so far.

D. RESULT & DISCUSSION

1. Result

The Hypothetical Learning Trajectory in learning multiplication and division that has been prepared is tested at the pilot experiment stage by the teacher in the class. HLT and Mathematical Tasks were tested in third grade X Elementary School, totaling 14 students. From the results of observations made during the learning process in class, there were responses from students that did not match the assumptions made in the HLT. This student response is the process of grouping LEGO students before carrying out the process of arranging LEGO to resemble the shape of a temple. Students do this to classify LEGO based on its color in accordance with the concept of place value, which has weight. For this reason, researchers carried out revisions to the HLT that had been prepared by providing activities to understand the concept of place value in subsequent learning. Figure 2 shows the process of grouping LEGOs based on color (place value) carried out by students at the pilot experiment stage.

Figure 2.

Documentation of LEGO grouping by students

The results of observations during the implementation of the pilot experiment led to revisions to the HLT that had been previously designed, especially in learning activities that were previously directly involved in the LEGO arranging process, adding LEGO grouping based on color. The improvements to learning activities include: (1) The teacher invites students to group LEGO based on colors that have place value weights; (2) The teacher names the numbers and asks students to show what LEGOs are needed to get the value of the numbers mentioned by the teacher. After the HLT improvements occurred, the researcher carried out a teaching experiment at Y Elementary School.

The teaching experiment stage is the HLT implementation stage, which has been improved according to the findings of the pilot experiment. The learning objectives at the teaching experiment stage are still the same as at the pilot experiment stage, namely: (1) Students can understand the concept of multiplication and division of integers through the context of Prambanan Temple; (2) Students can carry out multiplication and division of whole numbers in the context of Prambanan Temple. The learning activity designed is assembling and disassembling LEGO as a representation of the context of the Prambanan Temple, which is given to students. This activity is expected to direct students' thinking processes on the concept of multiplication and division operations. Documentation of the LEGO arrangement carried out by students in groups can be seen in Figure 3 below:

Figure 3.

LEGO Assembling Documentation

After students are asked to arrange LEGO according to the Prambanan Temple building as context, the LEGO used will be counted based on the color of the LEGO. Students are asked to group the LEGOs used according to colors that have an agreed place value weight. Each group has different LEGO construction which causes differences in the amount of LEGO used. Documentation of the results of assembling LEGOs resembling Prambanan Temple and filling in the Worksheet for each group can be seen in Table 2-4 below:

Table 2. LEGO Assembling and First group Worksheet Results on Multiplication Activities

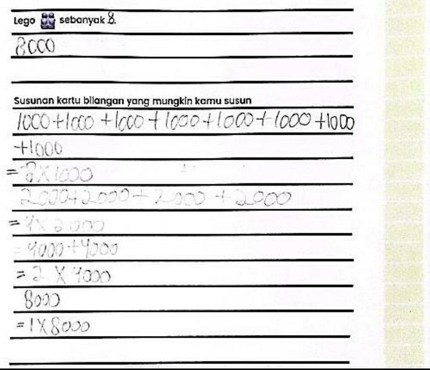
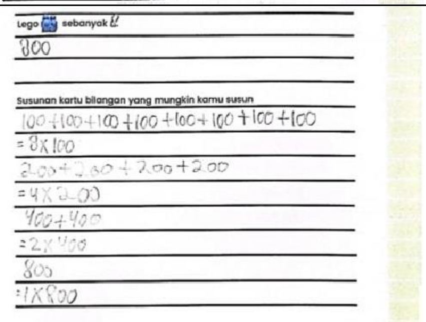
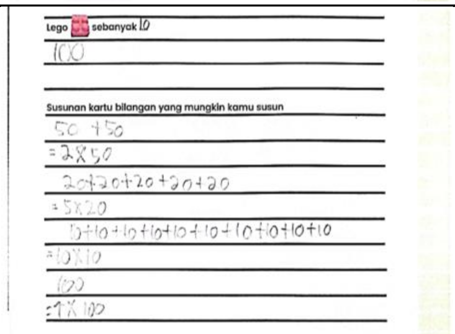
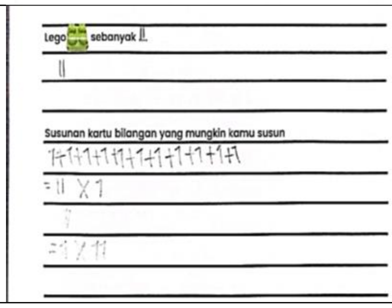
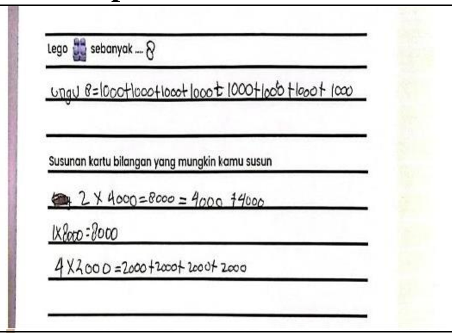
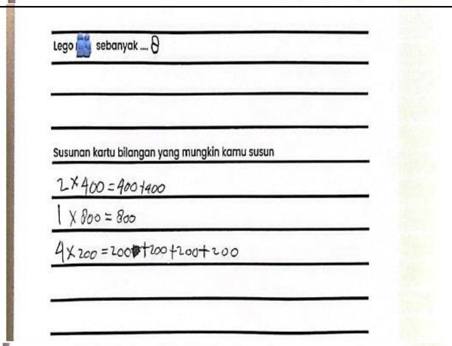
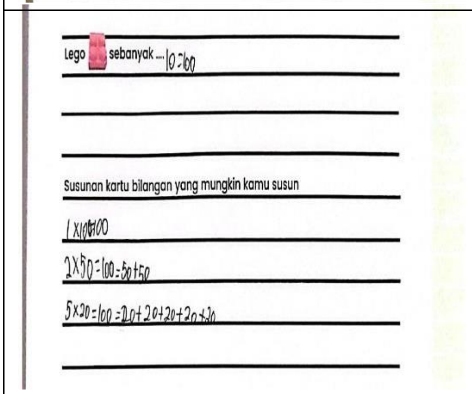
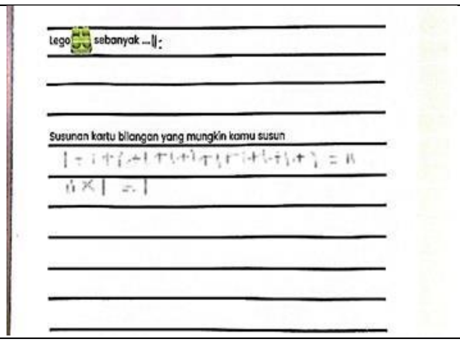
First Group	
	 Lego sebanyak 8. 8000 Susunan kartu bilangan yang mungkin kamu susun 1000+1000+1000+1000+1000+1000+1000 +1000 =8X1000 2000+2000+2000+2000 =4X2000 4000+4000 =2X4000 8000 =1X8000
	 Lego sebanyak 8 800 Susunan kartu bilangan yang mungkin kamu susun 100+100+100+100+100+100+100+100 =8X100 200+200+200+200 =4X200 400+400 =2X400 800 =1X800
	 Lego sebanyak 10 100 Susunan kartu bilangan yang mungkin kamu susun 50+50 =2X50 20+20+20+20+20 =5X20 10+10+10+10+10+10+10+10+10+10 =10X10 100 =1X100
	 Lego sebanyak 11 11 Susunan kartu bilangan yang mungkin kamu susun 1+1+1+1+1+1+1+1+1+1+1 =11 X 1 11 =1X11

Table 3. LEGO Assembling and Group 2 Worksheet Results on Multiplication Activities

Second Group 2	
	 Lego sebanyak ... 8 ungu 8=1000+1000+1000+1000+1000+1000+1000+1000 Susunan kartu bilangan yang mungkin kamu susun 2 X 4000 = 8000 = 4000 + 4000 1X8000 = 8000 4X2000 = 2000+2000+2000+2000
	 Lego sebanyak ... 8 Susunan kartu bilangan yang mungkin kamu susun 2X400 = 400+400 1 X 800 = 800 4X200 = 200+200+200+200
	 Lego sebanyak ... 10 = 100 Susunan kartu bilangan yang mungkin kamu susun 1 X100=100 2X50 = 100 = 50+50 5x20 = 100 = 20+20+20+20+20
	 Lego sebanyak ... 11 Susunan kartu bilangan yang mungkin kamu susun

Tabel 4. LEGO Assembling and Group 3 Worksheet Results on Multiplication Activities

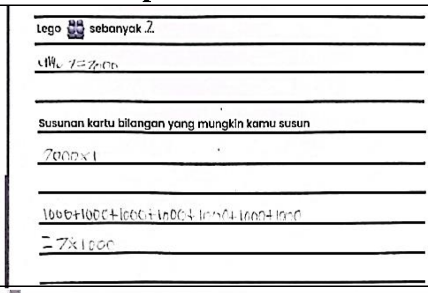
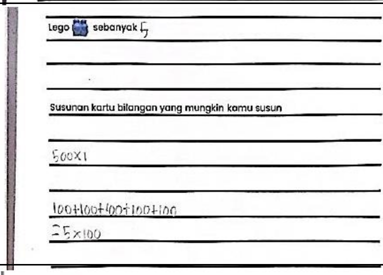
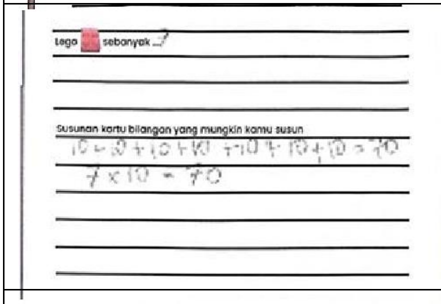
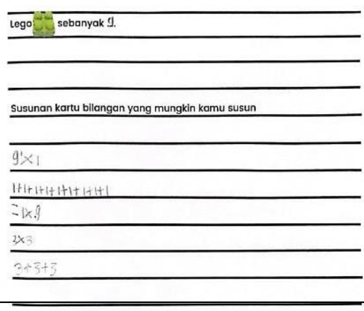
Third Group	
	
	
	
	

Table 2 shows the results of the LEGO arrangement for first group, where the total number of LEGOs used by this group was 8,911. In this case, first group used 8 Purple LEGOs (thousands), 8 Blue LEGOs (hundreds), 10 Pink LEGOs (dozens), and 11 Green LEGOs (ones). This number is the same as the LEGO used by group 2, namely 8,911 (Table 3). Even though the arrangement of the same number of LEGOs was used, the results of groups 1 and 2 were quite significantly different. Meanwhile, table 4 shows the results of arranging LEGO and filling in the worksheet for group 3. The number of LEGOs used by this group was 7,579. Group 3 used 7 Purple LEGOs (thousands), 5 Blue LEGOs (hundreds), 7 Pink LEGOs (dozens), and 9 Green LEGOs (ones).

Furthermore, for the division activity, researchers used LEGO stickers to encourage students to think mathematically when the division process was carried out. Because each group is given the same number, the results of filling in the worksheets for groups 1, 2, and 3 are the same. Documentation of the results of filling in the worksheet for student division activities can be seen in Figure 4 below:

Figure 4.

Student Worksheet Documentation in Division Activities

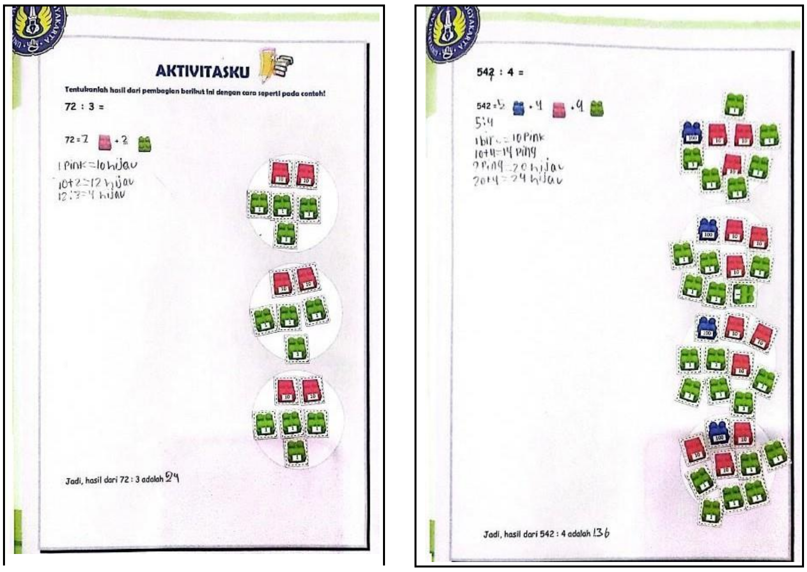


Figure 4 shows the division activities carried out by students. Students are given two division operations and then asked to carry out the division process with one object at a time. Because they are still in the initial introduction to the concept of division, the mathematical activities carried out by students occur manually and do not involve a complex division process. Students are asked to convert from larger LEGO place values to smaller LEGO place values.

Referring to the results of the analysis of the process of assembling LEGO and filling out worksheets in classroom learning, the picture of student learning trajectories can be described in several stages, namely, (1) Situational Stage. At this stage, students observe pictures of the Prambanan Temple and understand the problems that form the context of learning in class; (2) Referential Stage. At this stage, students group LEGO according to their color. Students do this to agree on the weight of the place value of each LEGO group according to its color; (3) Practical Stage. At this stage, students carry out LEGO building activities for learning multiplication and sticker sticking activities for learning division; (4) Mathematical stage. At this stage, students write down the mathematical form of the multiplication and division activities carried out; and (5) Formal Stage. At this stage, the teacher formulates and assigns the concepts of multiplication and division developed by students. A summary of the resulting Learning Trajectory can be seen in Figure 5 below:

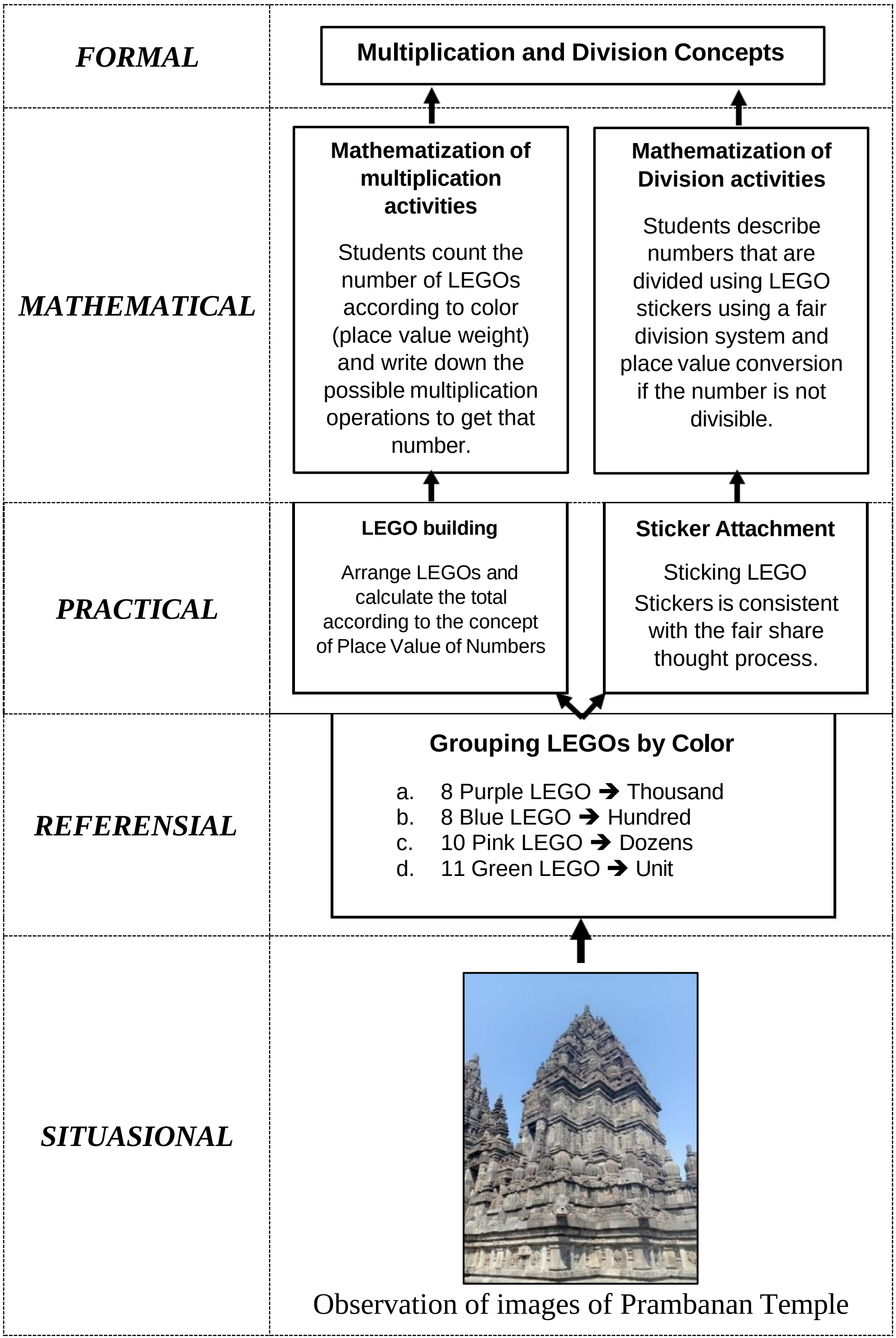


Figure 5. Learning Trajectory in the context of Prambanan Temple on multiplication and division material

2. Discussion

a. LEGO Building Results

Based on findings in the field, the results of LEGO construction carried out by students have several differences. As was done by groups 1 and 2, where even though the amount of LEGO used in assembling the LEGO was the same, the results of assembling the LEGO for the two groups were different. The number of LEGOs used by these two groups was 8 Purple LEGOs (thousands), 8 Blue LEGOs (hundreds), 10 Pink LEGOs (dozens), and 11 Green LEGOs (ones). A comparison of the results of building LEGO groups 1 and 2 can be seen in the illustration in Figure 6 below:

Figure 6.

Illustration of the LEGO First group and Group 2 assembling

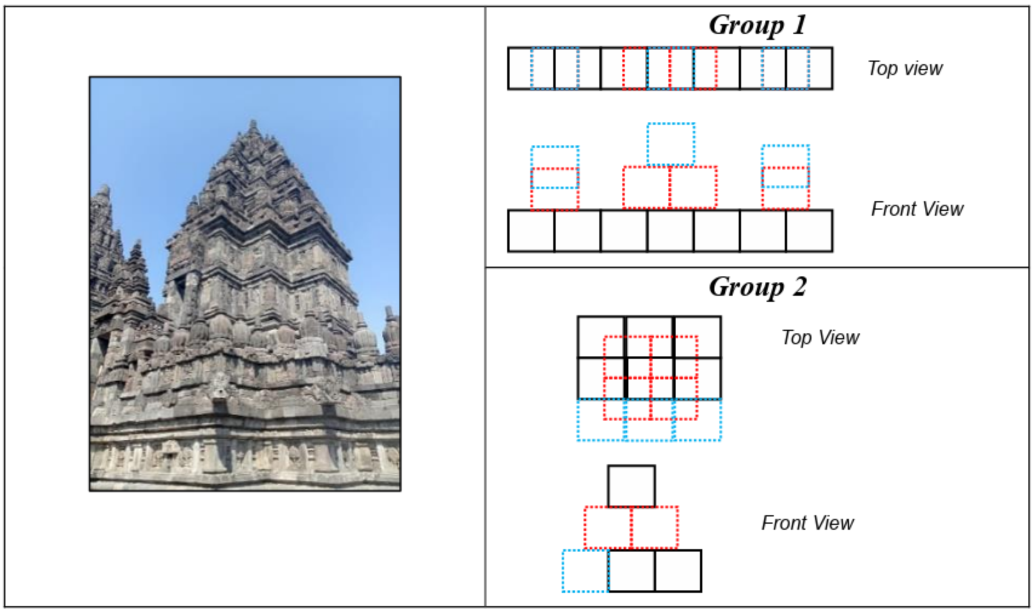


Figure 6 shows the differences in LEGO arrangement from groups 1 and 2. Group 2 built the Prambanan Temple building with a straight-line foundation. Then stack upwards. Meanwhile, Group 2 used square-shaped building foundations, which were then stacked systematically and tapered upwards. From a visual representation point of view, the results of group 2 are more like the Prambanan Temple building, which is arranged with a square foundation (all sides have the same length). Visual representation ability is one of the mathematical abilities in understanding visual objects to be represented in mathematical models (Garderen and Montague 2003). In line with this, the results of King (2014) research stated that visual representation abilities will have an impact on students' mathematical abilities.

However, from the perspective of learning tools, the use of LEGO can optimize students' divergent abilities. This divergent ability is facilitated by LEGO, which can be seen from the differences in the shape of LEGO buildings between groups, even though the building elements are the same (Chen, Chang, and Wu 2020). The research of Voss, Kruse, and Kent-Schneider (2022) explained that bringing out divergent abilities in students in learning mathematics is very useful for increasing students' creativity (Taylor et al. 2020; Sun, Wang, and Wegerif 2020). Thus, the choice of LEGO in preparing the learning trajectory has a positive impact on students.

b. Students' Multiplication Thinking Trajectories

In the process of multiplying, LEGO facilitates students to understand multiplication as repeated addition. This definition is a basic understanding that students must have before carrying out multiplications with more complicated and difficult numbers (Putri et al. 2023; Tillema 2013). In this research, it was found that students already understand multiplication as repeated addition as seen in the worksheet in Figure 7 below:

Figure 7.

Students' thinking trajectories during the multiplication process

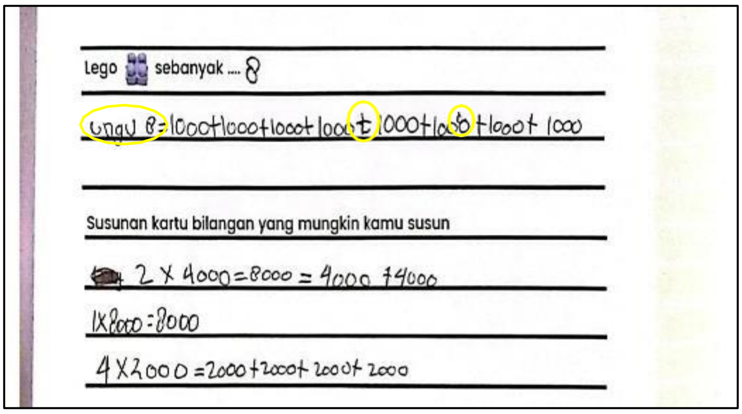


Figure 7 shows that the number of purple LEGOs used by students is 8. Students already understand that purple represents the place value of thousands, so students write the formula for adding 8 purple LEGOs as:

$$8000 = 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000$$

This formula is then explained in terms of the possible multiplication of 2 numbers that might be done to get the number "8000" are the following 3 possibilities:

- ⇨ $2 \times 4000 = 4000 + 4000 = 8000$
- ⇨ $4 \times 2000 = 2000 + 2000 + 2000 + 2000 = 8000$
- ⇨ $1 \times 8000 = 8000$

Conceptually, students already understand that the meaning of the multiplication "2×4000" is the result of adding 4000 by two, or in the mathematical formula, it becomes "$4000 + 4000$" which produces the number "8000". This contrasts with the results of research by MacDonald et al. (2018); Önal and Altiner (2022), which state that lower-grade students still cannot understand the concept of place value, which has a certain weight. This is refuted because, in this study, it was found that students were able to carry out multiplication operations by converting weighted place values (colors and place values) well. However, there are still several errors in written mathematical communication, especially in students' worksheets. In Figure 8, students use the words "8 purple", where what the students mean is "8000". Apart from that, students also make mistakes in writing the symbols " + " and "0", which can cause errors at the calculation stage.

According to Patahuddin, Ramful, and Greenlees (2015), mathematical communication errors made by students are often the cause of calculation errors and concluding when solving problems. Apart from that, mathematical communication errors can also have an impact on students' habits in writing mathematical symbols in the future (Wilson 2019). Therefore, mathematical communication errors made by students should be corrected by teachers from an early age so that they do not become students' habits, especially in mathematics learning, which will use more complicated mathematical symbols.

c. Student Division Thinking Trajectories

The division learning activities designed in this research were made as simple as possible considering that the research objects were lower class students (Third-Grades). The division process carried out by students is a fair distribution system, where students will count starting from the number that has the greatest place value weight. This process can be seen in Figure 8 below:

Figure 8.

Students' thinking trajectories during the multiplication process

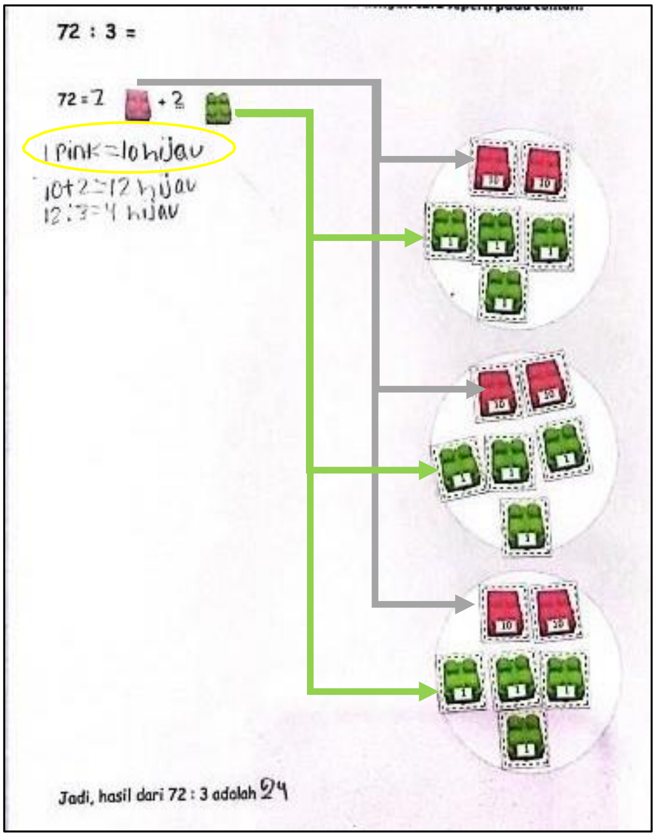


Figure 8 shows how students complete the division operation “72 ÷ 3”. The number 72 is then idealized with 7 Pink LEGOs and 2 Green LEGOs. Initially, students divided equally the number "70" which was idealized with pink LEGO stickers (Tens). These seven pink LEGO stickers were divided equally into three groups so that each group got two pink LEGO. After this process, one pink LEGO sticker remains. Students then convert these LEGO stickers into unit form, where 1 LEGO Pink sticker is the same as 10 LEGO Green stickers. This can be seen from the results of the student worksheet, which says "1 Pink = 10 Green". Because previously there were 2 Green LEGOs, this means that currently there are still 12 Green LEGOs which must be divided into three groups. In the same process, students divide 12 green LEGOs into three groups so that each group gets four green LEGOs. After that, students counted the number of LEGOs in each group and got the result 2 LEGO Pink and 4 LEGO Green, or mathematically 24. This shows that lower-grade students already understand the concept of place value, which has weight. This is contrary to the opinion of Önal & Altiner (2021), which states that students at the elementary school level still understand the concept of place value as the number of objects. A mathematical process carried out in stages like this can also help students improve number sense in carrying out division operations. This is because students will start dividing with base-10, no longer carrying out a division process that views the number to be divided as a whole number (Abon 2021; Kosko 2019). Solving numbers according to their place value will make it easier for students to carry out division operations with more complicated numbers later (Laurence 1991).

E. CONCLUSION

Based on the research results, it was concluded that learning using the Prambanan Temple context and LEGO learning media can help students understand the concepts of multiplication and division. The learning trajectory of the concept of multiplication and division in this research is a student learning trajectory obtained through a series of mathematical activities at several different stages of mathematical modeling. Student learning trajectories start from the situational stage, namely observing natural Prambanan Temple objects and understanding problems related to that context. In the referential stage, students' group LEGOs based on color (place value weight) to simplify the process of arranging LEGOs. Next, in practice, students carry out multiplication activities by arranging LEGO and division by sticking LEGO stickers. For the mathematical stage, students write down the mathematical form of the multiplication and division activities carried out. Finally, for the formal stage, the teacher formulates and confirms the concepts of multiplication and division developed by students.

Based on the conclusions obtained, several suggestions were made by the researcher, namely: (1) learning trajectory research in the context of Prambanan Temple and assisted by LEGO media helps students to understand the concept of multiplication and division in elementary schools so that similar research can be carried out to help students understand other mathematical concepts; and (2) learning planning that takes into account student responses is highly recommended by teachers so that the expected learning objectives can be achieved more optimally.

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